**Precalculus, Quarter 4, Unit 4.1**

**Matrices**

---

**Overview**

**Number of instructional days:** 11  
(1 day = 45–60 minutes)

**Content to be learned**

- Add, subtract, and use scalar multiplication with matrices and equivalent matrices.
- Understand when operations on matrices are appropriate or not based on dimensions (only square matrices can be inverted; restrictions on matrix addition and multiplication).
- Perform matrix multiplication including understanding that matrix multiplication is not commutative, but is associative and distributive.
- Solve 2 x 2 systems of equations using inverse matrices with and without technology.
- Solve 3 x 3 systems of equations using inverse matrices with technology.
- Represent and manipulate data using matrices.

**Mathematical practices to be integrated**

- Use appropriate tools strategically.
- Use appropriate technology to solve problems involving matrices.
- Look for and make use of structure.
- Evaluate work and make modifications or try a new approach, if necessary.
- Understand how multiplying by the identity matrix results in the same matrix both as IA and AI for any matrix A.

**Model with mathematics.**

- Apply mathematical results in the context of the problem situation and reflect on whether the results make sense.
- Interpret results in the context of the problem situation and reflect on the validity of the results.

**Essential questions**

- What are the advantages and disadvantages of solving matrix problems without using technology?
- How are matrices useful in representing real-world data?
- How can matrices be useful in the organization of data?
- Why are matrices useful in solving systems of equations?
- What real-life situations employ the use of matrices?
- What are the similarities and differences between the properties and operations used with real numbers and/or matrices?
Written Curriculum

Common Core State Standards for Mathematical Content

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Reasoning with Equations and Inequalities

A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater.)

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students learned commutative, associative, and identity properties of addition beginning in grade 1. In grade 3, the identity and commutative properties of multiplication were introduced, as well as the distributive property. In grade 6, the additive inverse property was introduced, and the multiplicative inverse property was introduced in grade 7.

Current Learning

Students add, subtract, and multiply matrices. They also use scalar multiplication, and they use inverse matrices to solve systems of equations. Students organize and represent data using matrices, and they make connections between and across the real number system and the matrix system. All this content is learned with and without technology.

Future Learning

In post secondary mathematics and physical science classes, vector analysis will be investigated further in the $n$-dimensional plane involving solutions to systems of equations.

Cumberland, Lincoln, and Woonsocket Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin
Additional Findings

*Beyond Numeracy* discusses the uses of matrices in systems of equations and also vectors (pp. 136–140).
Precalculus, Quarter 4, Unit 4.2
Modeling Vectors

Overview

Number of instructional days: 9 (1 day = 45–60 minutes)

Content to be learned

- Determine whether or not a quantity represents a vector.
- Recognize that vector quantities have both magnitude and direction; represent vectors as directed line segments.
- Use appropriate symbols for vectors and their magnitudes to include \( v, |v|, ||v||, v \).
- Find coordinates of a vector by subtracting the coordinates of an individual point from the coordinates of the terminal point.
- Solve problems involving velocity and other physical quantities modeled by vectors.
- Multiply a vector by a matrix of suitable dimensions to produce another vector.
- Represent matrices as transformations of vectors.
- Represent a system of linear equations as a single matrix equation in a vector variable.

Mathematical practices to be integrated

Reason abstractly and quantitatively.
- Attend to the meaning of quantities.
- Make sense of quantities in problem-solving situations.

Attend to precision.
- Use precise mathematical vocabulary, clear and accurate definitions, and symbols to communicate efficiently and effectively.
- Calculate and compute accurately (including using technology).

Model with mathematics.
- Relate what has been learned in mathematics to everyday life.

Essential questions

- What real-life situations give rise to the use of matrices/vectors?
- What are the requirements for a number to be considered a vector?
- What is the significance of representing number quantities as vectors?
- What are the similarities and differences between \( v, |v|, ||v||, v \)? In what situations are each of these vector forms used?
- Why are vectors characterized by both magnitude and direction?
- What are the similarities and differences between the absolute value of a vector and the norm of a vector?
- In what practical situations do vectors play an important role?
- What relationships exist between matrices and vectors?
- What are the advantages of solving a system of equations using both matrix and vector variables?
Written Curriculum

Common Core State Standards for Mathematical Content

<table>
<thead>
<tr>
<th>Vector and Matrix Quantities</th>
<th>N-VM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Represent and model with vector quantities.</strong></td>
<td></td>
</tr>
<tr>
<td>N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ( \mathbf{v} ), (</td>
<td>\mathbf{v}</td>
</tr>
<tr>
<td>N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</td>
<td></td>
</tr>
<tr>
<td>N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reasoning with Equations and Inequalities</th>
<th>A-REI</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.</td>
<td></td>
</tr>
</tbody>
</table>

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Cumberland, Lincoln, and Woonsocket Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin

52
6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In geometry students learned the distance formula and worked with distance on the coordinate plane. In algebra 2, students worked with absolute value and solved absolute value equations. In a previous unit in precalculus, students learned to plot polar coordinates.

Current Learning

Students learn that vectors represent magnitude and direction and solve problems involving velocity and other physical quantities modeled by vectors. Students use appropriate symbols for vectors and their magnitudes to include $v$, $|v|$, $||v||$, $v$. Students relate vectors to matrices by representing transformations and systems of linear equations using both vectors and matrices.

Future Learning

Vector analysis will introduce students to parametric equations, graphing relations, and the application of derivatives in motion. Applications of the derivative will include optimization, and related rates will incorporate vector analysis.

Additional Findings

According to *Beyond Numeracy*, vectors are used with quantities that require two or more dimensions to be specified. They play a primary role in linear algebra as well as in many other areas of applied mathematics (pp. 136–140).
Precalculus, Quarter 4, Unit 4.3
Vector Operations

**Overview**

<table>
<thead>
<tr>
<th>Number of instructional days:</th>
<th>12</th>
<th>(1 day = 45–60 minutes)</th>
</tr>
</thead>
</table>

**Content to be learned**

- Add vectors by the following methods: end-to-end, component wise, and parallelogram rule.
- Use and apply the magnitude of a sum and the sum of the magnitudes as part of vector analysis.
- Compute the magnitude and direction of the sum of two vectors within this form.
- Represent vector subtraction and scalar multiplication both graphically and component-wise.
- Calculate the dot product of vectors.
- Compute the additive inverse of a vector $v$.
- Compute the magnitude of a scalar multiple using $|c|v = ||cv||$.
- Compute direction of $cv$ when $c > 0$ or $c < 0$.

**Mathematical practices to be integrated**

- Use appropriate tools strategically.
- Use tools such as a graphing calculator to perform vector operations.
- Look for and make use of structure.
  - Look for a pattern or structure.
  - Evaluate work and make modifications or try a new approach, if necessary.
- Look for and express regularity in repeated reasoning.
  - Determine the reasonableness of results when performing calculations.

**Essential questions**

- How do properties of real numbers apply to vectors?
- What real-life situations employ the use of vectors?
- What similarities exist between the real numbers and vector quantities pertaining to the graphical representation?
- How are vectors used to simplify operations involving magnitude and direction?
**Common Core State Standards for Mathematical Content**

**Vector and Matrix Quantities**

<table>
<thead>
<tr>
<th><strong>Perform operations on vectors.</strong></th>
<th><strong>N-VM</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N-VM.4</strong> (+) Add and subtract vectors.</td>
<td></td>
</tr>
<tr>
<td>a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</td>
<td></td>
</tr>
<tr>
<td>b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</td>
<td></td>
</tr>
<tr>
<td>c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</td>
<td></td>
</tr>
</tbody>
</table>

| **N-VM.5** (+) Multiply a vector by a scalar. | |
| a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v}_x, \mathbf{v}_y) = (c\mathbf{v}_x, c\mathbf{v}_y)$. | |
| b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$). | |

| **Perform operations on matrices and use matrices in applications.** | |
| **N-VM.11** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | |
| **N-VM.12** (+) Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. | |

**Common Core Standards for Mathematical Practice**

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to...
identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\left(\frac{y - 2}{x - 1}\right) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1), (x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In elementary grades, properties of real numbers such as commutative, associative, and identities were introduced. In later grades, prime factorization, divisibility rules, and the distributive property were taught. In geometry, students learned the distance formula and worked with distance on the coordinate plane. In Algebra 2, students worked with absolute value and solved absolute value equations. In a previous unit in precalculus, students plotted polar coordinates.

Current Learning

In this unit, students expand their knowledge of vectors through the operations of addition, subtraction, and scalar multiplication, both graphically and component-wise. Students find the dot product of vectors as well as the additive inverses. Students compute the magnitude of a scalar multiple using $|c|v = ||cv||$ and compute direction of $cv$ when $c > 0$ or $c < 0$. 
Future Learning

Vector analysis will introduce the student to parametric equations, graphing relations, and the application of derivatives in motion. Applications of the derivative will include optimization, and related rates will incorporate vector analysis.

Additional Findings

According to *Beyond Numeracy*, vectors are used with quantities that require two or more dimensions to be specified. They play a primary role in linear algebra as well as in many other areas of applied mathematics (pp. 136–140).
Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned
• Use Cavalieri’s Principle to derive formulas for different solids.

Mathematical practices to be integrated
Construct viable arguments and critique the reasoning of others.
• Derive volume formulas using Cavalieri’s Principle and justify why this works.
Attend to precision.
• Use known formulas to connect concepts and build new formulas for volume.

Essential questions
• How does Cavalieri’s Principle help calculate the volume of irregular shapes in the real world?
• In what situations would it be necessary to implement Cavalieri’s Principle to determine volume?
### Written Curriculum

#### Common Core State Standards for Mathematical Content

<table>
<thead>
<tr>
<th>Geometric Measurement and Dimension</th>
<th>G-GMD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Explain volume formulas and use them to solve problems</strong></td>
<td></td>
</tr>
<tr>
<td>G-GMD-1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s Principle, and informal limit arguments.</td>
<td></td>
</tr>
<tr>
<td>G-GMD.2 (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.</td>
<td></td>
</tr>
<tr>
<td><strong>Visualize relationships between two-dimensional and three-dimensional objects</strong></td>
<td></td>
</tr>
<tr>
<td>G-GMD.4 Identify the shapes of two dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</td>
<td></td>
</tr>
</tbody>
</table>

#### Common Core Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

---

Cumberland, Lincoln, and Woonsocket Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin
Clarifying the Standards

Prior Learning

In grades 1–2, students demonstrated conceptual understanding of length, height, perimeter, and area using nonstandard units and models or manipulatives to represent polygons. In grade 3, the conceptual understanding extended to perimeter of polygons and area of rectangles on grids. Students expressed all measures using appropriate units. In grade 4, the perimeter and area extended to polygons or irregular shapes on grids using formulas. In grade 5, perimeter and area of right triangles were studied as well as volume of rectangular prisms (cubes). In grade 6, perimeter and area of quadrilaterals and all types of triangles were studied. Volume of rectangular prisms within a problem-solving context was introduced, and segment relationships within circles (radius, diameter, circumference) were studied. In grade 7, students determined area of circles, perimeter, and area of composite figures (any quadrilateral, triangle, and parts of a circle). The surface area of rectangular prisms was studied. Volumes of rectangular prisms, triangular prisms, and cylinders were emphasized. In grade 8, surface area and volume were extended to triangular prisms, cylinders, pyramids, and cones. In grades 9–10, perimeter, circumference, area, surface area, and volume of 2- and 3-dimensional figures, including composite figures, were developed.

Beginning in grade 3, students demonstrated conceptual understanding of visualization and spatial reasoning by copying, comparing, and drawing models of triangles, squares, rectangles, rhombi, trapezoids, hexagons, and circles. The study of 3-D model building of rectangular prisms began. In grade 4, octagons were studied as well as 2-D model building of rectangular prisms. In grade 5, 2-D or 3-D representations were introduced for triangular prisms, cones, cylinders, and pyramids. In grade 7, sketching 3-dimensional solids and drawing nets of rectangular and triangular prisms and nets of cylinders and pyramids were used to determine the technique for finding surface area. In grades 9–10, dynamic geometric software was used to generate 3-D objects from 2-D perspectives (or vice versa). Alternatively, this was done by solving related problems. In grade 8, students learned the volume formulas for regular shapes. Students also learned and applied the Pythagorean theorem.

Current Learning

Students use the Pythagorean Theorem to calculate various lengths needed to apply Cavalieri’s Principle. In advanced mathematical studies, students solve problems calculating volume using arcs, sectors, and circle segments using Cavalieri’s Principle. Students find area and volume of non-regular shapes and partition them into rectangles, squares, prisms, trapezoids, and cylinders. Students apply Cavalieri’s Principle to various regular and irregular shapes to calculate volume.

Future Learning

In calculus, these ideas will be extended into the concepts of finding area under a curve, area between curves, volumes of cross-sectional solids, net area, net accumulation, the definite integral, rectangular approximation method, trapezoidal approximation method, Simpson’s rule, and the limit definition of the integral using Riemann Sums. In many career fields—including but not limited to architecture, surveying, the arts, engineering, and computer technology (CAD)—perimeter, area, surface area, and volume applications will be employed.

Cumberland, Lincoln, and Woonsocket Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin
**Additional Findings**

Research on this content can be found in the following references:

- *Beyond Numeracy* (pp. 18–23, 129–132, and 251–256);
- *Science for All Americans* (pp. 134–135);
- *Principles and Standards for School Mathematics* (pp. 308–318 and 320–323); and

This research states that the area of a figure can be found by partitioning it into triangles or rectangles. Trigonometric ratios of similar triangles are equal. Many students understand square units as things to be counted rather than as subdivisions of the plane.