Geometry, Quarter 3, Unit 3.1
Similar Triangles

Overview

Number of instructional days: 11 (1 day = 45–60 minutes)

Content to be learned

• Use proportional reasoning to identify similar triangles; determine scale factor, and use scale factor to find missing sides of triangles.
• Solve problems using the geometric mean between two numbers and parts of right triangles.
• Identify similar triangles by AA, SAS, and SSS.
• Use proportions to find lengths of segments formed by parallel lines that intersect two or more transversals.
• Solve real-world problems involving similar triangles.

Essential questions

• How do you use proportions to find side lengths of similar triangles?
• How do you show that two triangles are similar?
• How do you identify corresponding parts of similar triangles?

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

• Use prior material and properties of mathematics to solve more complex problems involving deductive and inductive reasoning.

Model with mathematics.

• Draw conclusions based on relationships and models.

• How can you use proportions, other than for similar triangles, to solve a real-world problem?
• How are geometric mean and similarity related?
• What happens when two or more parallel lines intersect other lines?
### Written Curriculum

**Common Core State Standards for Mathematical Content**

<table>
<thead>
<tr>
<th>Similarity, Right Triangles, and Trigonometry</th>
<th>G-SRT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understand similarity in terms of similarity transformations</strong></td>
<td></td>
</tr>
<tr>
<td>G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</td>
<td></td>
</tr>
<tr>
<td>G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</td>
<td></td>
</tr>
<tr>
<td><strong>Prove theorems involving similarity</strong></td>
<td></td>
</tr>
<tr>
<td>G-SRT.4 Prove theorems about triangles. <em>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</em></td>
<td></td>
</tr>
<tr>
<td>G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</td>
<td></td>
</tr>
</tbody>
</table>

**Common Core Standards for Mathematical Practice**

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 7, students analyzed proportional relationships and used them to solve problems. In grade 8, they were introduced to similarity through physical models.

Current Learning

In this unit, students use ratios and proportions to prove whether or not two polygons are similar and to find unknown side lengths. Students show that triangles are similar based on the relationships of two or three pairs of corresponding parts (AA, SSS, SAS). Students use geometric mean to discuss similarity in right triangles.

Future Learning

Students will use the content of this unit in measurement and trigonometric applications.

Additional Findings

According to Principles and Standards for School Mathematics, transformations can also be used to help students understand similarity (p. 236).

According to A Research Companion to Principles and Standards for School Mathematics, under the right conditions, children of all ages can apply similarity transformations to shapes, and middle school students develop spatial abilities after completing similarity tasks (p. 160).
Geometry, Quarter 3, Unit 3.2

Trigonometry

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Use the properties of special right triangles to solve problems.
- Make and defend conjectures to solve problems using trigonometric ratios (sine, cosine, tangent).
- Apply trigonometric ratios to real-world problems.
- (+) Find unknown measurements in triangles that are not right triangles, using the Laws of Sines and Cosines.

Mathematical practices to be integrated

Model with mathematics.

- Relate what has been learned in mathematics to everyday life.
- Interpret results in the context of a problem.

Use appropriate tools strategically.

- Use various tools including technology to help solve problems.

Attend to precision.

- Use labels and units of measure correctly.
- Calculate and compute accurately.

Essential questions

- How do you use trigonometry to find a missing side length or angle measure in a right triangle?
- How do trigonometric ratios relate to similar right triangles?
- How is the process to solve for a side different than the process to solve for an angle of a right triangle?
- How are the sine, cosine, and tangent ratios similar and different?
- (+) When do you use the Law of Sines vs. the Law of Cosines?
- How can you use the properties of special right triangles as a shortcut to solve problems?
Written Curriculum

Common Core State Standards for Mathematical Content

<table>
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<th><strong>Similarity, Right Triangles, and Trigonometry</strong></th>
<th><strong>G-SRT</strong></th>
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</table>

**Define trigonometric ratios and solve problems involving right triangles**

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

**Apply trigonometry to general triangles**

G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Common Core Standards for Mathematical Practice

4 **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 8 (8.G6, 8.G7, 8.G8), students were introduced to and applied the Pythagorean Theorem. In the previous unit of this course, students made and defended conjectures to solve problems using the Pythagorean Theorem.

Current Learning

In this unit, students make and defend conjectures to solve problems using trigonometric ratios, the Law of Sines, and the Law of Cosines. Students also apply trigonometric ratios and the Laws of Sines and Cosines to solve real-world problems.

Future Learning

Students will study the unit circle, modeling, and simple trigonometry in future courses (F-TF.5). In precalculus, students will continue to use unit circle trigonometry, graphing trigonometric functions, trigonometric identities, and inverse trigonometric functions (F-TF.3.4.6.7.9, F-IF.7). In calculus, students will continue to use trigonometry. Also in the algebra 2 course, students use the special right triangle formulas to prove the unit circle ($x^2 + y^2 = 1$).
Additional Findings

According to *Principles and Standards for School Mathematics*, students in grades 9–12 will use right triangle trigonometry to solve a range of practical problems (p. 313).

According to *A Research Companion to Principles and Standards for School Mathematics*, as students develop their understanding of trigonometric ratios, they should use models to simulate them (p. 166).
Geometry, Quarter 3, Unit 3.3
Perimeter and Area

Overview

Number of instructional days: 5
(1 day = 45–60 minutes)

Content to be learned

- Solve problems using circumference and area of polygons/circles.
- Determine how changes in dimensions affect perimeter, circumference, and area.
- Find the perimeter and area of composite shapes both algebraically and graphically.
- Solve area and perimeter problems using dynamic software when available.
- Use trigonometry to find the area of a triangle i.e., $A=\frac{1}{2}ab\sin(C)$.

Mathematical practices to be integrated

Model with mathematics.

- Relate what has been learned in mathematics to everyday life.

Attend to precision.

- Use labels and units of measure correctly.
- Calculate and compute accurately, including technology.

Essential questions

- How are the circumference and area of a circle related?
- How are the formulas for the areas of polygons related?
- How is the area of a polygon affected by changing its dimensions?
- How is the area of a figure different from its perimeter?
- How can you use trigonometry to find the area of a regular polygon?
### Written Curriculum

#### Common Core State Standards for Mathematical Content

**Expressing Geometric Properties with Equations**

<table>
<thead>
<tr>
<th>G-GPE</th>
<th>Use coordinates to prove simple geometric theorems algebraically</th>
<th>[Include distance formula; relate to Pythagorean theorem]</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-GPE.7</td>
<td>Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*</td>
<td></td>
</tr>
</tbody>
</table>

**Geometric Measurement and Dimension**

<table>
<thead>
<tr>
<th>G-GMD</th>
<th>Explain volume formulas and use them to solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-GMD.1</td>
<td>Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.</td>
</tr>
</tbody>
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**Similarity, Right Triangles, and Trigonometry**

<table>
<thead>
<tr>
<th>G-SRT</th>
<th>Apply trigonometry to general triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-SRT.9</td>
<td>(+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</td>
</tr>
</tbody>
</table>

#### Common Core Standards for Mathematical Practice

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
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Clarifying the Standards

Prior Learning

In grade 6, students learned the formulas for area of right triangles, other triangles, special quadrilaterals, and polygons. They used these formulas to solve real-world problems. In grade 7, students learned the formulas for area and circumference of a circle and used those formulas to solve real-world problems. Also in seventh grade, they used surface area of three-dimensional shapes to solve problems.

Current Learning

Students solve problems using the perimeter of a figure, circumference of a circle, and area of a figure. They determine how the change of a dimension affects the perimeter, area, and circumference. Students also find perimeter and area of composite shapes.

Future Learning

In precalculus, students will use area formulas to give informal arguments to explain volume formulas (G.GMD.2).

Additional Findings

According to Benchmarks for Science Literacy, by the end of fifth grade, students should know that length can be thought of as unit lengths joined together, area as a collection of unit squares, and volume as a set of unit cubes (p. 223).

According to Principles and Standards for School Mathematics, instructional programs from prekindergarten through grade 12 should enable all students to understand and use formulas for the area, surface area, and volume of geometric figures (p. 320).
Geometry, Quarter 3, Unit 3.4
Volume

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone.
- Solve problems involving volume of pyramids, prisms, cylinders, cones, and spheres.
- Use geometric shapes and their properties to describe objects.
- Identify shapes of two-dimensional cross sections of three-dimensional objects.
- Identify three-dimensional objects generated by rotations of two-dimensional objects.
- Apply geometric concepts in modeling situations involving area, and volume

Mathematical practices to be integrated

- Construct viable arguments and critique the reasoning of others.
- Use prior material and properties of mathematics to solve more complex problems involving deductive and inductive reasoning.
- Model with mathematics.
- Relate what has been learned in mathematics to everyday life.
- Use appropriate tools strategically.
- Use various tools, including technology, to help solve problems.

Essential questions

- Why is volume an important concept and where would you use it in the real world?
- How do changes in the dimensions of a solid affect its volume?
- How is surface area related to volume?
- Why are the formulas for the volume of solids different?
- How does a horizontal cross-section of a three-dimensional object differ from a vertical cross-section of that same object?
- What happens when you rotate a two-dimensional figure around a given line to sweep out a solid of revolution?
Written Curriculum

Common Core State Standards for Mathematical Content

**Geometric Measurement and Dimension**

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</tr>
<tr>
<td>G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</td>
</tr>
</tbody>
</table>

**Visualize relationships between two-dimensional and three-dimensional objects**

| G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |

**Modeling with Geometry**

<table>
<thead>
<tr>
<th>Apply geometric concepts in modeling situations</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
</tr>
<tr>
<td>G-MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).</td>
</tr>
<tr>
<td>G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).</td>
</tr>
</tbody>
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**Common Core Standards for Mathematical Practice**

3. **Construct viable arguments and critique the reasoning of others.**

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Clarifying the Standards

Prior Learning

In grade 6, students found volumes for right rectangular prisms and applied those formulas to solve real-world problems. In grade 7, students described two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right triangular prisms and right triangular pyramids. Also in grade 7, students used right prism volume formulas to solve real-world problems. In grade 8, students learned the formulas for volumes of cones, cylinders, and spheres and used them to solve real-world problems.

Current Learning

In this unit, students solve real-world problems both algebraically and graphically using volume of pyramids, prisms, cylinders, cones, and composite shapes. When possible, students use dynamic geometric software or other technology to enhance learning. Students compare surface area, area, and volume of similar solids.

Cumberland, Lincoln, and Woonsocket Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin
**Future Learning**

In grade 12, students will give an informal argument for the formulas for the volume of a sphere and other solid figures using Cavalieri’s Principle.

**Additional Findings**

According to *Principles and Standards for School Mathematics*, instructional programs should enable all students to analyze properties and determine attributes of two- and three-dimensional objects (p. 308).

According to *Benchmarks for Science Literacy*, by the end of twelfth grade, students should know that there are formulas for calculating the surface areas of regular shapes. Students should know that when the linear size of a shape changes by some factor, its area changes in proportion to the square of the factor (p. 225).